Question 1
Which of the following intervals would be the best to use to find the instantaneous rate of change of \( f(x) = x^2 + 3x - 2x\) at \( x = 3\)?

A. \( 2.5 \leq x \leq 3 \)
B. \( 3 \leq x \leq 3.5 \)
C. \( 2.5 \leq x \leq 3.5 \)
D. \( 2 \leq x \leq 3 \)

Question 2
Which of the following is the best estimate for the rate of change of \( f(x) = \sin(x)\) at \( x = 0\)?

A. 0
B. 1
C. -1
D. 0.5

Question 3
Find \( \lim_{x \to 2} f(x)\) given the following information:
• \( f(x) \) is continuous at \( x = 2 \)
• \( f(2) = 4 \)
• \( \lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) \)

A. Nonexistent
B. Not enough information
C. \( \lim_{x \to 2} f(x) = 2 \)
D. \( \lim_{x \to 2} f(x) = 4 \)

Question 4

What is the average rate of change of \( f(x) = x^3 \) on the interval \( 0 \leq x \leq 3 \)

A. \(-9\)
B. \(27\)
C. \(9\)
D. \(-27\)

Question 5

Which of the following is NOT required for \( f(x) \) to be continuous at \( x = c \)?

A. \( f(x) \) is not a piecewise function
B. \( f(c) \) exists
C. \( \lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x) \)
D. \( \lim_{x \to c} f(x) \) exists

Question 6

\[ \lim_{x \to -8} e^x + 4 \]

A. DNE
B. 4
C. \(\infty\)
D. \(-4\)

Question 7

For the function below, identify the following limit:
\[
\lim_{x \to 3} f(x)
\]

\[
f(x) = \begin{cases} 
-x^2 + 7, & x \leq 3 \\
x - 5, & x \geq 3
\end{cases}
\]

A. \(-2\)
B. DNE
C. 2
D. 0

Question 8

Identify the following limit using the graph below. Note that the graphs \( f(x) \) and \( g(x) \) are labelled by \( f \) and \( g \) respectively.

\[
\lim_{x \to 2} \frac{f(x)}{g(x)}
\]

A. \(-\frac{1}{2}\)
B. 2
C. \(-2\)
D. \(-\frac{2}{3}\)

Question 9
The function \( f \) is defined over the real numbers. This table gives a few values of \( f \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.9</td>
<td>9.62</td>
</tr>
<tr>
<td>2.99</td>
<td>9.85</td>
</tr>
<tr>
<td>2.999</td>
<td>9.99</td>
</tr>
<tr>
<td>3.001</td>
<td>9.99</td>
</tr>
<tr>
<td>3.01</td>
<td>9.85</td>
</tr>
<tr>
<td>3.1</td>
<td>9.62</td>
</tr>
</tbody>
</table>

A. 2.9  
B. 3  
C. 9.9  
D. 10  
E. DNE

Question 10

We want to find \( \lim_{x \to 0} x \sin(x) \). Direct substitution and other algebraic methods don't seem to work. Look at the graph of \( f(x) = \frac{x}{\sin(x)} \), we can estimate that the limit is equal to 1.

\[
\lim_{x \to 0} \frac{x}{\sin(x)} = 1
\]

To prove that  \( \lim_{x \to 0} \frac{x}{\sin(x)} = 1 \), we can use the squeeze theorem.

Kendrick suggested that we use the functions \( g(x) = 1 \) and \( h(x) = e^x + 1 \) in order to apply the squeeze theorem.
Does Kendrick’s suggestion seem to be correct?

A. Yes, Kendrick’s suggestion seems to be correct.
B. No, Kendrick’s suggestion is incorrect because it’s not true that one function is always below \( f \) and one function is always above it for \( x \)-values near 0.
C. No, Kendrick’s suggestion is incorrect because it’s not true that the limits of \( g \) and \( h \) are both equal to 1.

Question 11

Let \( g(x) = \frac{x^2 - x - 12}{x - 4} \) when \( x \neq 4 \). \( g \) is continuous for all real numbers. Find \( g(4) \).

A. 4  
B. 7  
C. -4  
D. -3

Question 12

There are the graphs of functions \( f \) and \( g \). Dashed lines represent asymptotes. Which functions are continuous over the interval \([-6, 6]\)?

Choose all answers that apply.
Question 13

Consider graphs A, B, and C. The dashed lines represent asymptotes. Which graphs agree with this statement?

\[ + \lim_{x \to 0^+} f(x) = \infty \]

Choose all answers that apply.
Question 14

Let \( f(x) = \frac{3}{x} \). Select the correct description of the one-sided limits of \( f \) at \( x=0 \).

A. \( + \lim_{x \to 0^+} f(x) = +\infty \) and \( -\lim_{x \to 0^-} f(x) = +\infty \)
B. \( + \lim_{x \to 0^+} f(x) = -\infty \) and \( -\lim_{x \to 0^-} f(x) = -\infty \)
C. \( + \lim_{x \to 0^+} f(x) = -\infty \) and \( -\lim_{x \to 0^-} f(x) = +\infty \)
D. \( + \lim_{x \to 0^+} f(x) = -\infty \) and \( -\lim_{x \to 0^-} f(x) = -\infty \)

Question 15

Let \( g \) be a continuous function on the closed interval \([1,5]\). A few values of \( g \) are given in this table:

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x )</td>
<td>-1</td>
<td>4</td>
<td>9</td>
<td>15</td>
</tr>
</tbody>
</table>

Which intervals must contain a solution to \( g(x) = 3 \)?
A. [-1, 2]
B. [2, 4]
C. [4, 5]
D. None of the above

Question 16

The table gives selected values of the function $f$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>-10</th>
<th>-5</th>
<th>0</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>11</td>
<td>7</td>
<td>30</td>
<td>-15</td>
</tr>
</tbody>
</table>

Can we use the intermediate value theorem to say the equation $f(x) = 10$ has a solution where $0 \leq x \leq 15$?

A. No, since we don’t know if the function is continuous on that interval.
B. No, since 10 is not between $f(0)$ and $f(5)$.
C. Yes, both conditions for using the intermediate value theorem have been met.

Question 17

If $\lim \limits_{x \to 0} f(x)$ exists,

A. $\lim \limits_{x \to -\infty} f(x)$ exists.
B. $f(x)$ must be continuous at all $x$ values.
C. $f(x)$ must be continuous at all $x=0$.
D. $f(0)$ exists and $\lim \limits_{x \to 0} f(x) = f(0)$.
E. We cannot conclude any of the other answers.
Answer Key

1. C
2. B
3. D
4. C
5. A
6. B
7. A
8. C
9. D
10. C
11. B
12. A
13. C
14. B
15. A
16. A
17. E