

AP Calculus AB Unit 3 — Differentiation – Composite, Implicit, and Inverse Functions Practice Test

Question 1

If $x^3 + y^3 = 6xy$, find the derivative through implicit differentiation.

A. $y' = \frac{2y - x^2}{y^2 - 2x}$

B. $y' = \frac{y - x^2}{y^2 - x}$

C. $y' = \frac{2y + 2x}{x - y}$

D. $y' = \frac{3y - 6x^2}{6y^2 - 3x}$

E. $y' = \frac{2y + x^2}{y^2 + 2x}$

Question 2

Find the derivative of $f(x) = 10^{2\sqrt{x}}$.

A. $f'(x) = 10^{2\sqrt{x}} * x^{\frac{-1}{2}}$

B. $f'(x) = 10^{x - \frac{1}{2}} * \ln 10$

C. $f'(x) = 10^{2\sqrt{x}} * 2x^{\frac{-1}{2}} * \ln 10$

D. $f'(x) = 10^{2\sqrt{x}} * \ln 10 * x^{\frac{-1}{2}}$

E. $f'(x) = \ln 10 * x^{\frac{-1}{2}}$

Question 3

$f(x) = 6^{5x^2 - 7x}$. Find the derivative.

A. $f'(x) = (30x^2 - 42x) * \ln 6$

B. $f'(x) = 6^{5x^2 - 7x} * (10x - 7) * \ln 6$

C. $f'(x) = 6^{10x - 7} * \ln 6$

D. $f'(x) = \ln 6 * (10x - 7)$

E. $f'(x) = 6^{5x^2 - 7x} * (10x - 7)$

Question 4

$f(x) = (4x^3 - 9x^2 + 4x - 8)^2$. Using the chain rule for derivatives, find $f'(x)$.

- A. $f'(x) = 2(4x^3 - 9x^2 + 4x - 8)^2 * (12x^2 - 18x + 4)$
- B. $f'(x) = (12x^2 - 18x + 4)^2$
- C. $f'(x) = 8x^5 - 18x^4 + 8x^2 - 16$
- D. $f'(x) = 2(4x^3 - 9x^2 + 4x - 8)^2$
- E. $f'(x) = 12x^2 - 18x + 4$

Question 5

Find the derivative of the function of the circle $x^2 + y^2 = 25$.

- A. 1
- B. $\frac{-x}{y}$
- C. $\frac{-y}{x}$
- D. $\frac{x}{y}$
- E. 1

Question 6

Find the derivative of the function $\ln y = \ln x^6$.

- A. undefined
- B. $y' = \frac{6x}{y}$
- C. $y' = \frac{6y}{x}$
- D. $y' = \frac{y}{6x}$
- E. $y' = \frac{x}{6y}$

Question 7

$f(x) = \sin(x^2)$. Find $f'(x)f'(x)$.

- A. $f'(x) = 2x \cos(x^2)$
- B. $f'(x) = \cos(x^2)$

- C. $f'(x) = -2x \cos(x^2)$
- D. $f'(x) = 2x \cos(x)$
- E. $f'(x) = 2x \sin(x^2)$

Question 8

Find the derivative of the exponential function, $y = e^{\sec 3\theta}$.

- A. $y' = e^{\sec(3\theta)\tan(3\theta)}$
- B. $y' = 3e^{\sec(3\theta)\tan(3\theta)}$
- C. $y' = \sec(3\theta)\tan(3\theta)$
- D. $y' = 3e^{\sec 3\theta} * \sec(3\theta)\tan(3\theta)$
- E. $y' = 3e^{\sec(3\theta)}$

Question 9

Let f and g be inverse functions. The following table lists a few values of f, g , and f' .

x	$f(x)$	$g(x)$	$f'(x)$
2	1	9	$\frac{1}{3}$
9	2	10	$\frac{1}{12}$

Find $g'(2)$.

- A. 10
- B. 11
- C. 12
- D. 13
- E. 14

Question 10

Let $g(x) = 3 - 2x - x^3$ and let f be the inverse function of g . Notice that $g(1) = 0$. Find $f'(0)$.

- A. $\frac{-1}{2}$
- B. $\frac{-1}{3}$
- C. $\frac{-1}{4}$
- D. $\frac{-1}{5}$
- E. $\frac{-1}{6}$

Question 11

Given $f(x) = \arctan(3x)$, **find** $f'(x)$.

- A. $\frac{3}{1-9x^2}$
- B. $\frac{-3}{1-9x^2}$
- C. $\frac{3}{1+9x^2}$
- D. $\frac{-3}{1+9x^2}$

Question 12

Jacob tried to find the derivative of $x^9 \cdot \sqrt{x}$. **Here is his work:**

Step 1: This is the product of x^9 and \sqrt{x} . So we should use the product rule.

Step 2:

$$\frac{d}{dx}[x^9 \cdot \sqrt{x}] = \frac{d}{dx}[x^9] \cdot \frac{d}{dx}[\sqrt{x}]$$

Step 3: Finding the derivatives of the factors:

$$\frac{d}{dx}[x^9] = 9x^8$$

$$\frac{d}{dx}[\sqrt{x}] = \frac{1}{2}x^{-\frac{1}{2}}$$

Step 4: Putting it all together:

$$\begin{aligned} & \frac{d}{dx}[x^9 \cdot \sqrt{x}] \\ &= \frac{d}{dx}[x^9] \cdot \frac{d}{dx}[\sqrt{x}] \\ &= 9x^8 \cdot \frac{1}{2} \cdot x^{-\frac{1}{2}} \\ &= \frac{9}{2}x^{\frac{15}{2}} \end{aligned}$$

Is Jacob's work correct? If not, what's his mistake?

- A. Jacob's work is correct.
- B. Step 1 is incorrect. Jacob should have used a different rule and not the product rule.
- C. Step 2 is incorrect. Jacob didn't state the correct product rule.
- D. Step 3 is incorrect. Jacob didn't differentiate \sqrt{x} correctly.

Question 13

$$g(x) = x(5x^4 + 7x^3 - x + 6)$$

How would you rewrite $g(x)$ so it can be differentiated using the power rule?

- A. $x(5x^4 + 7x^3) - x(x - 6)$
- B. $5x^5 + 7x^4 - x^2 + 6x$

- C. $\frac{x}{(5x^4+7x^3-x+6)^{-1}}$
 D. This is not possible.

Question 14

$y = x^2 \tan(2x^5)$. Find $\frac{dy}{dx}$.

- A. $2x \tan(2x^5) + x^2 \sec^2(2x^5)$
 B. $2x \tan(2x^5) + 10x^6 \sec^2(2x^5)$
 C. $14x^8 \sec^2(2x^7)$
 D. $2x^7 \sec^2(x) + 14x^6 \tan(x)$

Question 15

$f(x) = \arctan(3x)$. Find $f'(x)$.

- A. $\frac{3}{1-9x^2}$
 B. $\frac{-3}{1-9x^2}$
 C. $\frac{3}{1+9x^2}$
 D. $\frac{-3}{1+9x^2}$

Question 16

How many derivatives can you take of a function?

- A. Only 3
 B. Only 2
 C. Only 1
 D. Until you can no longer derive it

Question 17

Let $y = 4 \sin(3x)$. Find $\frac{d^2y}{dx^2}$.

- A. $-36 \sin(3x)$

- B. $\frac{4}{9} \cos(3x)$
- C. $-4 \sin(3x)$
- D. $12 \cos(3x)$

Question 18

Let $y^4 - 2x = 5$.

What is the value of $\frac{d^2y}{dx^2}$ at the point $(-2,1)$?

- A. $\frac{3}{4}$
- B. $\frac{-3}{5}$
- C. $\frac{2}{5}$
- D. $\frac{-3}{4}$

Question 19

Find $f'''(x)$ when $f(x) = 10x^5 + 3x^4 - 5x^3 + 2x^2 - 10x + 6$

- A. $200x^3 + 36x^2 - 30x + 4$
- B. $600x^2 + 72x - 26$
- C. $600x^2 + 72x - 30$
- D. $600x^2 + 72x - 36$

Answer Key

1. A
2. D
3. B
4. A
5. B
6. C
7. A
8. D
9. C
10. D
11. C
12. C
13. B
14. B
15. C
16. D
17. A
18. D
19. C