AP Calculus AB Unit 4 — Contextual Applications of Differentiation Practice Test

Question 1

A particle is moving in a straight path with a constant initial velocity. The particle is then subjected to a force causing a time-dependent acceleration given as a function of time: $a(t)=(a+b)t$

After 10 seconds, the particle has a velocity equal to k **meters-per-second. Find the initial velocity in terms of the constants** k **,** a **, and** b

Units are all in S.I. (meters, seconds, meters-per-second, etc.)

A. $v_o=\frac{k}{a+b}-50a$
B. $v_o=\frac{k}{a}+b$
C. $v_o=b-5(a+k)$
D. $v_o=k-50(a+b)$
E. $v_o=\frac{b}{a}+k$

Question 2

The position of a particle as a function of time is given below:

$$x(t)=\frac{t^3}{3}-3t^2+8t$$

At what values of t **does the particle change direction?**

A. $t=3\wedge t=1$
B. $t=2\wedge t=4$
C. $t=0\,only$
D. $t=2\,only$
E. $t=3\,only$

Question 3

A gun sends a bullet straight up with a launch velocity of 220 ft/s. It reaches a height of $s=220t-16t^2$ **after** t **seconds. What is its velocity 500 ft into the air?**

A. $-128.1\wedge 128.1$
B. $158.1\wedge -158.1$

C. $138.1 \wedge -138.1$

Question 4

A projectile is shot up from a platform 5 m above the ground with a velocity of 150 m/s. Assume that the only force acting on the projectile is gravity that produces a downward acceleration of 9.8 m/s^2. Find the velocity as a function of t .

A. $v=-15t+9.8$
B. $v=-5t+9.8$
C. $v=-9.8t+150$
D. $v=-9.8t+5$
E. $v=-5t+150$

Question 5

A right triangle has sides of length x and y which are both increasing in length over time such that:

$x(t)=2t$

$y(t)=4t^2$

Find the rate at which the angle θ opposite $y(t)$ is changing with respect to time.

A. $\frac{d\theta}{dt}=\frac{4t}{1-t^2}$
B. $\frac{d\theta}{dt}=\sin(t)+\frac{1}{1+t}$
C. $\frac{d\theta}{dt}=\frac{2}{1+4t^2}$
D. $\frac{d\theta}{dt}=t+\frac{4}{1+2t^2}$
E. $\frac{d\theta}{dt}=\cos(t)-\frac{1}{1+t}$

Question 6

A tank is being filled with a liquid. The function V gives the volume of liquid in the tank, in liters, after t minutes.

What is the best interpretation for the following statement?

The value of the derivative at V at $t=1$ is equal to 2 .

A. After 1 minute, the tank was being filled at a rate of 2 liters.
B. After 1 minute, the tank had 2 liters of liquid.
C. After 1 minute, the tank was being filled at a rate of 2 liters per minute.
D. During the first minute, the tank was being filled at a rate of 2 liters per minute.

Question 7

A weight that is attached to the end of a spring is pulled and then released. The function H gives its height, in centimeters, after t seconds.

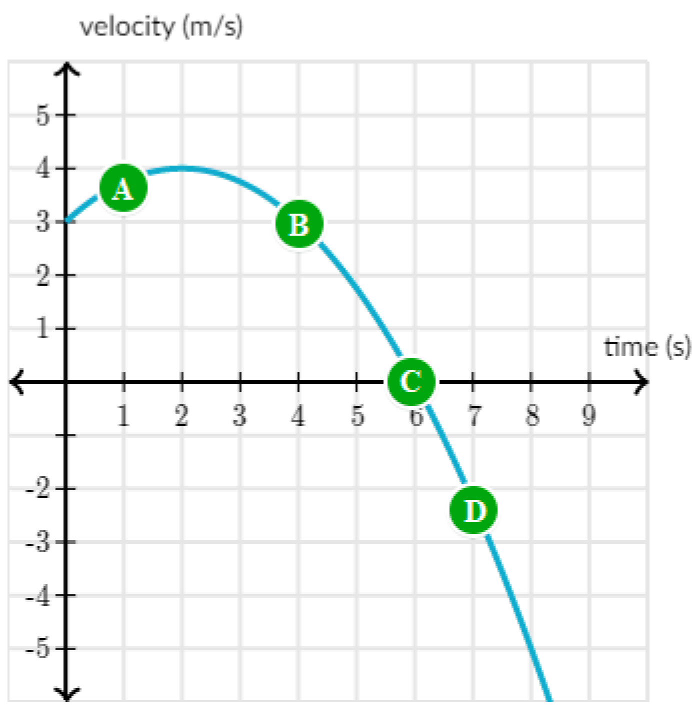
What is the best interpretation for the following statement?

$H'(0)=3$

A. When the weight is released, its height is 3 centimeters.
B. When the weight is released, its mass is increasing at a rate of 3 grams per second.
C. When the weight is released, its height is increasing at a rate of 3.
D. When the weight is released, its height is increasing at a rate of 3 centimeters per second.

Question 8

An object is moving along a line. The following graph gives the object's velocity over time.



Which point on the graph is neither speeding up nor slowing down?

- **A.** Point A
- **B.** Point B
- **C.** Point C
- **D.** Point D

Question 9

Nora uploaded a funny video on her website, which rapidly gains views over time. The following function gives the number of views t days after Nora uploaded the video:

$V(t)=100\cdot e^{0.4t}$

What is the instantaneous rate of change of the number of views 4 days after the video was uploaded?

- **A.** 198 views
- **B.** 198 views per day
- **C.** 495 views
- **D.** 495 views per day

Question 10

Consider the following problem:

The radius $r(t)$ of the base of a cylinder is increasing at a rate of 1 meter per hour and the height $h(t)$ of the cylinder is decreasing at a rate of 4 meters per hour. At a certain instant t_0, the base radius is 5 meters and the height is 8 meters. What is the rate of change of the volume $V(t)$ of the cylinder at that instant?

Find the values of $r(t_0), h'(t),$ and $\frac{dV}{dt}$.

- **A.** $r(t_0)=1; h'(t)=-4; \frac{dV}{dt}=8$
- **B.** $r(t_0)=5; h'(t)=-4; \frac{dV}{dt}=1$
- **C.** $r(t_0)=5; h'(t)=-4; \frac{dV}{dt}=not\ given$
- **D.** $r(t_0)=1; h'(t)=-4; \frac{dV}{dt}=not\ given$

Question 11

Tom was given this problem:

The side $s(t)$ **of a square is decreasing at a rate of** 2 **kilometres per hour. At a certain instant** t_0 **, the side is** 9 **kilometres. What is the rate of change of the area** $A(t)$ **of the square at that instant?**

Which equation should Tom use to solve the problem?

- **A.** $A(t)=4\cdot s(t)$
- **B.** $A(t)=[s(t)]^3$
- **C.** $A(t)=[s(t)]^2$
- **D.** $[A(t)]^2=[s(t)]^2+[s(t)]^2$

Question 12

The differentiable functions x **and** y **are related by the following equation:**

$\sin(y)=-5x$

Also, $\frac{dy}{dt}=10$ **. Find** $\frac{dx}{dt}$ **when** $y=-\pi$ **.**

- **A.** 0
- **B.** 1
- **C.** 2
- **D.** -1

Question 13

The radius of a circle is decreasing at a rate of 6.5 **meters per minute. At a certain instant, the radius is 12 meters. What is the rate of change of the area of the circle at that instant (in square meters per minute)?**

- **A.** -42.25π
- **B.** -156π
- **C.** -288π
- **D.** -144π

Question 14

One diagonal of a rhombus is decreasing at a rate of 7 **centimeters per minute and the other diagonal of the rhombus is increasing at a rate of** 10 **centimeters**

per minute. At a certain instant, the decreasing diagonal is 4 centimeters and the increasing diagonal is 6 centimeters. What is the rate of change of the area of the rhombus at that instant (in square centimeters per minute)?

- **A.** 16
- **B.** 1
- **C.** -1
- **D.** -16

Question 15

The surface area of a sphere is increasing at a rate of 14π square meters per hour.

At a certain instant, the surface area is 36π square meters.

What is the rate of change of the volume of the sphere at that instant (in cubic meters per hour)?

- **A.** 21π
- **B.** $\left(\sqrt{14\pi}\right)^3$
- **C.** 36π
- **D.** $\frac{7}{12}$

Question 16

The local linear approximation to the function g at $x=6$ is $y=-3x+4$. What is the value of $g(6)+g'(6)$?

- **A.** -17
- **B.** -18
- **C.** -19
- **D.** -20

Question 17

Let f be a differentiable function with $f(2)=-3$ and $f'(2)=-4$. What is the value of the approximation of $f(1.9)$ using the function's local linear approximation at $x=2$?

- **A.** -2.9
- **B.** -2.8
- **C.** -2.7

D. -2.6

Question 18

Find $\lim_{x \to 0} \frac{e^x - \cos x}{4 \sin x}$ **.**

A. $\frac{1}{4}$

B. $\frac{e-1}{4}$

C. $\frac{-1}{2}$

D. The limit doesn't exist.

Question 19

Find $\lim_{x \to \infty} \frac{x^4}{3x^2 - 7x}$ **.**

A. 0

B. $\frac{1}{3}$

C. $\frac{2}{3}$

D. ∞

Answer Key

1. D
2. B
3. A
4. C
5. C
6. C
7. D
8. C
9. B
10. C
11. C
12. C
13. B
14. C
15. A
16. A
17. D
18. A
19. D