AP Calculus AB Unit 5 — Analytical Applications of Differentiation Practice Test

Question 1

Let f be the function given by f(x) = x^3. What are all values of c that satisfy the conclusion of the Mean Value Theorem on the closed interval [-1,2]?

A. 0 only
B. 1 only
C. $\sqrt{3}$ only
D. -1 and 1

Question 2

Let $f(x)=2^x-\sin(\pi x)$ **. Below is Rafael's attempt to write a formal justification for the fact that the equation** $f'(x)=\frac{1}{4}$ **has a solution where** $-2<x\leftarrow 1$ **;**

Is Rafael's justification complete? If not, why?

Rafael's justification:

Exponential and trigonometric functions are differentiable and continuous at all points in their domain, and $-2 \leq x \leq -1$ is within f's domain.

So, according to the mean value theorem, $f'(x) = \frac{1}{4}$ must have a solution somewhere in the interval $-2 < x < -1$.

A. Yes, Rafael's justification is complete.
B. No, Rafael didn't establish that the average rate of change of f over $[-2,-1]$ is equal to $\frac{1}{4}$.
C. No, Rafael didn't establish that f is differentiable.

Question 3

Given a function $g(x)$ **, if** $g''(x)=0$ **for a certain value of** x **, then** $g(x)$ **has ______________________ at x.**

A. an inflection point
B. a maximum
C. a minimum
D. a critical point

Question 4

Let $h(x)=e^{2x-6}-e$. Where does h have critical points? Choose all answers that apply.

- **A.** $x=0$
- **B.** $x=\frac{5}{2}$
- **C.** $x=3$
- **D.** h has no critical points.

Question 5

Given a function, $f(x)$, if $f'(x)<0$ over a certain interval, then $f(x)$ is ________________ over that interval.

- **A.** decreasing
- **B.** concave up
- **C.** increasing
- **D.** concave down

Question 6

Let g be a function defined for all real numbers except for $x=2$.

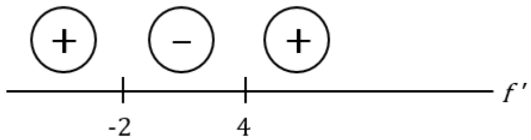
Also let g' , the derivative of g , be defined as $g'(x)=\frac{x^2}{(x-2)^3}$.

On which intervals is g is increasing?

- **A.** $(-\infty,0)$ and $(0,2)$
- **B.** $(-\infty,0)$ and $(2,\infty)$
- **C.** $(2,\infty)$ only
- **D.** $(0,2)$ only
- **E.** The entire domain of g

Question 7

Use the sign chart for $f'(x)$. There is/are…



A. a local maximum at $x=-2$.
B. a local minimum at $x=-2$ and a local maximum at $x=4$.
C. a local maximum at $x=-2$ and a local minimum at $x=4$.
D. no extrema.

Question 8

Let f be a polynomial function and let f' , its derivative, be defined as $f'(x)=x^4(x-2)(x+3)$. At how many points does the graph of f have a relative maximum?

A. None
B. One
C. Two
D. Three

Question 9

What is the maximum value of $f(x)=x^3-3x^2-1$ on the interval $[-3,2]$?

A. 0
B. -1
C. 2
D. 5

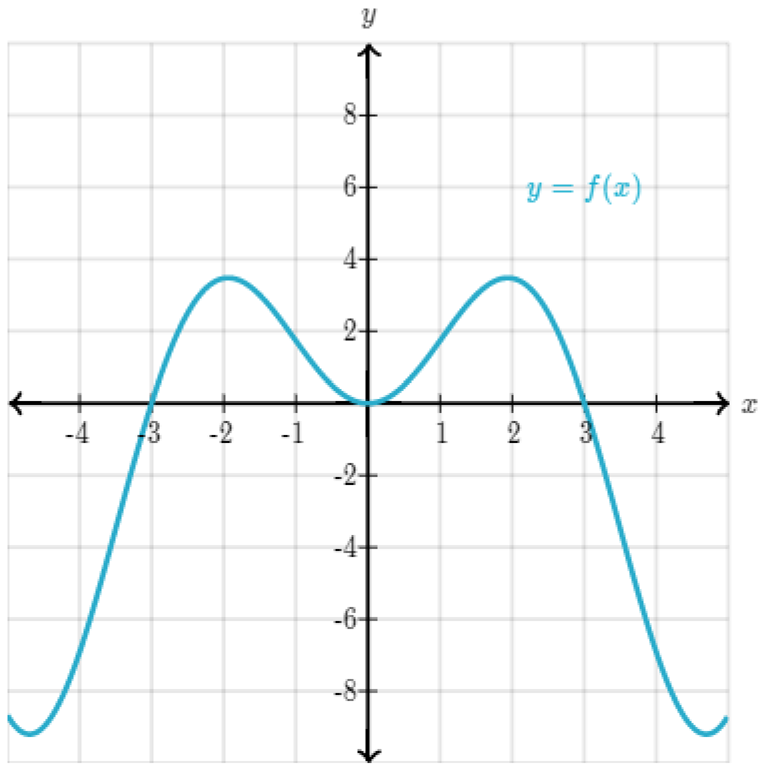
Question 10

Let $g(x)=2x^3-21x^2+60x$. What is the absolute maximum value of g over the closed interval $[0,6]$.

A. 36
B. 52
C. 25
D. 42

Question 11

Function f is graphed.



Select all the intervals where $f'(x)<0$ and $f''(x)>0$.

Choose all answers that apply.

A. $-4<x\leftarrow 3$
B. $-3<x\leftarrow 2$
C. $0<x<1$
D. None of the above

Question 12

The concavity of a function is described by its ____________________.

A. first derivative
B. second derivative
C. third derivative
D. expression

Question 13

Let h be a twice differentiable function, and let $h(-4)=-3, h'(-4)=0,$ and $h''(-4)=0$. What occurs in the graph of h at the point $(-4,-3)$?

A. $(-4,-3)$ is a minimum point.
B. $(-4,-3)$ is a maximum point.
C. There's not enough information to tell.

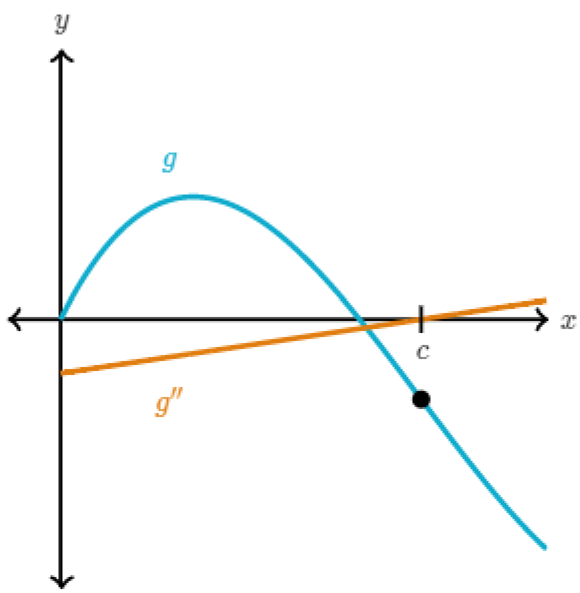
Question 14

Let f be a twice differentiable function, and let $f(2)=3, f'(2)=0$ and $f''(2)=5$. What occurs in the graph of g at the point $(2,3)$?

A. $(2,3)$ is a minimum point.
B. $(2,3)$ is a maximum point.
C. There's not enough information to tell.

Question 15

The twice differentiable function g and its second derivative g'' are graphed.



What is an appropriate calculus-based justification for the fact that g has an inflection point at $x=c$?

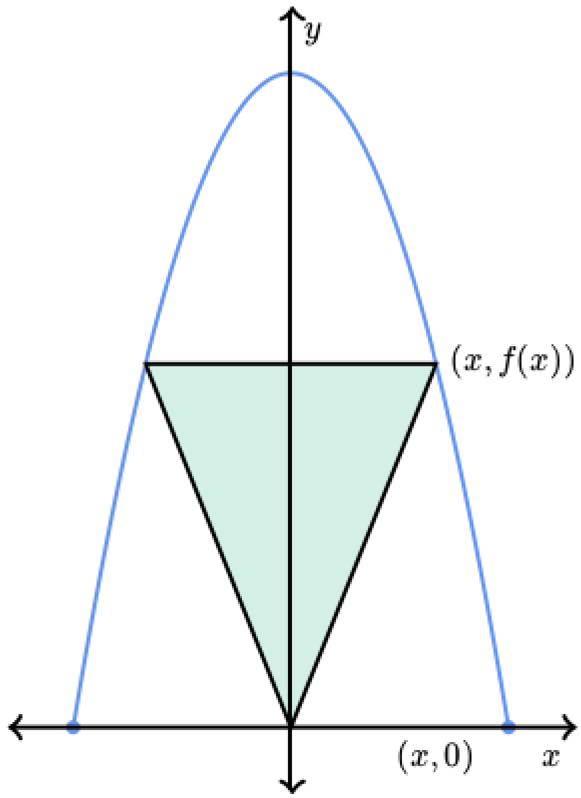
A. The graph of g changes the direction of its concavity at $x=c$.
B. g'' crosses the x -axis from below it to above it at $x=c$.
C. g'' is increasing at $x=c$.

Question 16

Consider the function $f(x)=9-x^2$ for $f(x)\geq 0$ only.

The shaded region is an isosceles triangle formed by joining the points $(0,0),(x,f(x))$, and $(-x,f(x))$, where $0\leq x\leq 3$.

What is the area of the largest triangle that satisfies the stated conditions?



A. $2\sqrt{2}$
B. $3\sqrt{2}$
C. $3\sqrt{3}$
D. $6\sqrt{3}$
E. None of these

Question 17

Kendall tried to find all the equations of vertical lines tangent to the curve given by $x^2+2xy^2=25$. This is her solution:

Step 1: Finding an expression for $\frac{dy}{dx}$.

$$\frac{dy}{dx} = \frac{-x - y^2}{2xy}$$

Step 2: Forming a system of equations.

$$\begin{cases} x^2 + 2xy^2 = 25 \\ 2xy = 0 \\ -x - y^2 \neq 0 \end{cases}$$

Step 3: Solving the system.

$x = -5, x = 0,$ and $x = 5$

Is Kendall's solution correct? If not, at which step did she make a mistake?

- **A.** The solution is correct.
- **B.** Step 1 is incorrect.
- **C.** Step 2 is incorrect.
- **D.** Step 3 is incorrect.

Question 18

The derivative of a function f **is given by** $f'(x) = \ln \vee x \vee \cdot x$ **. On which interval is the graph of** f **concave up? (Use a graphing calculator)**

- **A.** $-1 < x < 0$ and $x > 1$
- **B.** $x > 0$
- **C.** $x < 0$
- **D.** $x \leftarrow 0.368$ and $x > 0.368$
- **E.** All real numbers

Question 19

The derivative of a function g is given by $g'(x) = 3\sin(x) + \ln(x)$.

How many relative extremum points does the graph g have on the interval $1 < x < 5$? (Use a graphing calculator)

A. One
B. Two
C. Three
D. Four

Answer Key

1. B
2. B
3. D
4. D
5. A
6. C
7. C
8. B
9. B
10. B
11. D
12. B
13. C
14. A
15. B
16. D
17. D
18. D
19. A