

AP Calculus AB :Unit 8 — Applications of Integration Practice Test

Question 1

What is the average value of \sqrt{x} on the interval $[5, 12]$?

- A. $\frac{\sqrt{12^3} - \sqrt{5^3}}{21}$
- B. $\frac{\sqrt[3]{12^2} - \sqrt[3]{5^2}}{21}$
- C. $\frac{\sqrt{12} - \sqrt{5}}{2}$
- D. $\frac{\sqrt{12} + \sqrt{5}}{2}$

Question 2

Divya received the following problem:

A particle moves in a straight line with velocity $v(t) = \sqrt{3t-1}$ meters per second, where t is time in seconds. At $t=2$, the particle's distance from the starting point was 8 meters in the positive direction. What is the particle's position at $t=7$ seconds?

Which expression should Divya use to solve the problem?

- A. $8 + \int_0^7 v(t) dt$
- B. $8 + v'(7)$
- C. $v(7)$
- D. $8 + \int_2^7 v(t) dt$

Question 3

The population of a town grows at a rate of $r(t)$ people per year (where t is time in years). At $t=3$, the town's population was 1000 people.

What does $1000 + \int_3^8 r(t) dt = 1500$ mean?

- A. The average rate at which the population grew between $t=3$ and $t=8$ is 1500 people per year.
- B. The town's population grew by 1500 people between $t=3$ and $t=8$.
- C. The town's population grew at a rate of 1500 people per year at $t=8$.
- D. At $t=8$, the town's population was 1500 people.

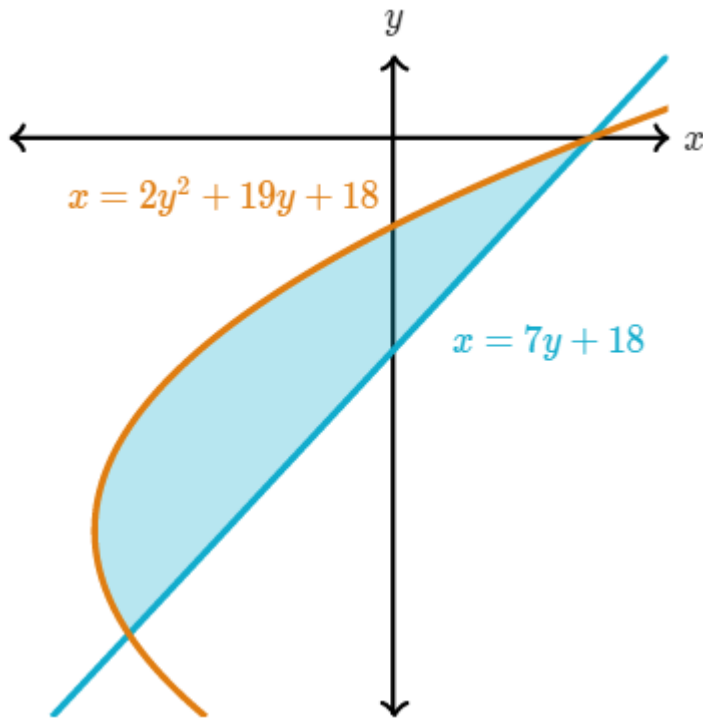
Question 4

What is the area of the region between the graphs of $f(x) = 8x + 6$ and $g(x) = x - x^2$ from $x = -6$ to $x = -1$?

- A. $\frac{125}{6}$
- B. $\frac{485}{6}$
- C. $\frac{1195}{6}$
- D. $\frac{643}{6}$

Question 5

The curves $x = 7y + 18$ and $x = 2y^2 + 19y + 18$ are graphed.

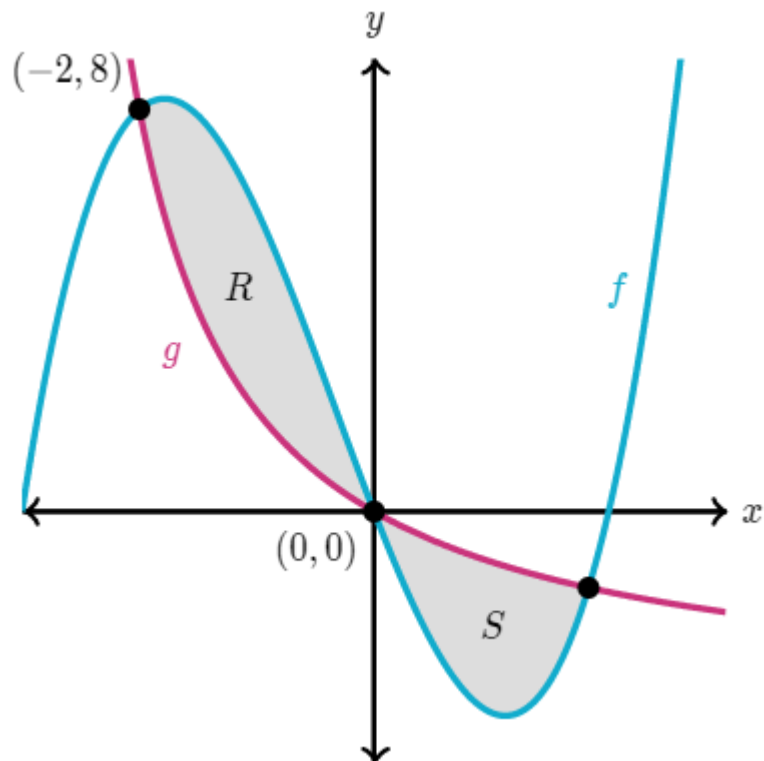


Which expression represents the area bounded by the curves?

- A. $\int_{-6}^0 (-2y^2 - 12y) dy$
- B. $\int_{-6}^0 (2y^2 + 26y + 36) dy$
- C. $\int_{-4.75}^{18} (-2y^2 - 12y) dy$
- D. $\int_{-4.75}^{18} (2y^2 + 26y + 36) dy$

Question 6

Let $f(x) = x^3 + x^2 - 6x$ and $g(x) = \frac{-4x}{x+3}$. Let R and S be two regions enclosed by the graphs of f and g as shown in the graph.

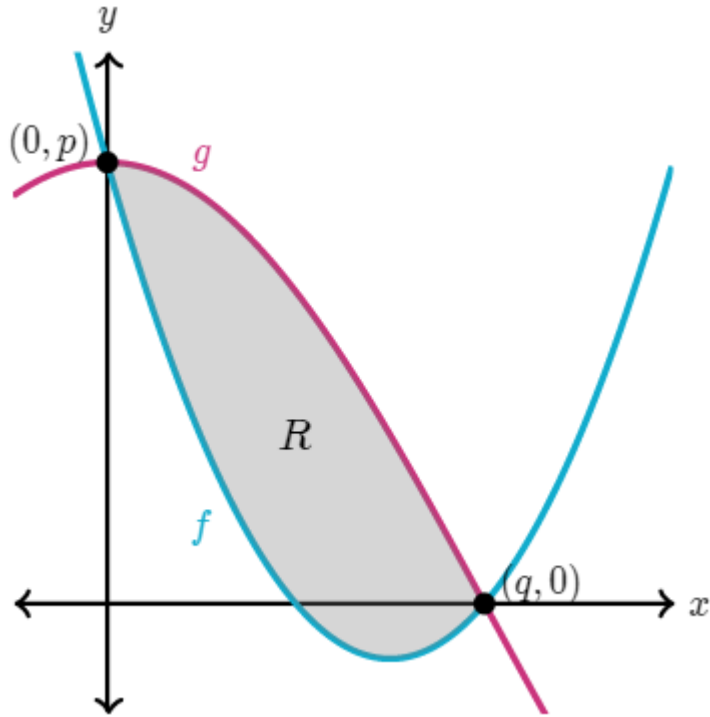


Find the sum of the areas of regions R and S . Round your answer to three decimal places.

- A. Sum of the areas ≈ 9.078
- B. Sum of the areas ≈ 10.078
- C. Sum of the areas ≈ 11.078
- D. Sum of the areas ≈ 12.078

Question 7

Let R be the region enclosed by the graphs of functions f and g .

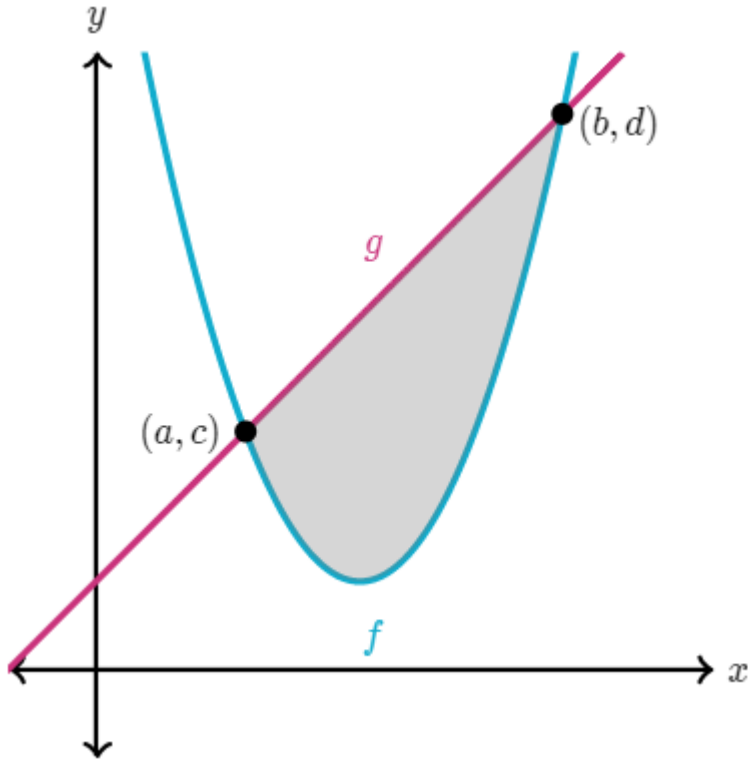


Region R is the base of a solid whose cross sections perpendicular to the x -axis are squares. Which one of the definite integrals gives the volume of the solid?

- A. $\int_0^q [g(x) - f(x)] dx$
- B. $\int_0^q [g(x) - f(x)]^2 dx$
- C. $\int_0^q [g(x) - f(x)]^2 dx$
- D. $\int_0^q [g(x) - f(x)] dx$

Question 8

The base of a solid S is the region bounded by the graphs of the functions f and g shown below.

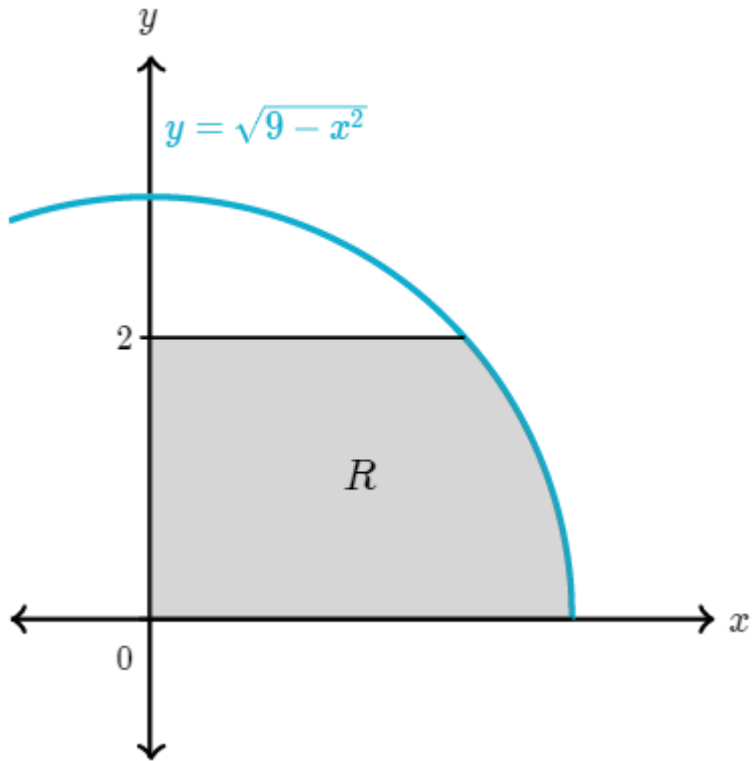


Cross-sections perpendicular to the x -axis are semi-circles. Which one of these integrals represents the volume of solid S ?

- A. $\frac{\pi}{8} \int_a^b [g(x) - f(x)]^2 dx$
- B. $\frac{\pi}{8} \int_c^d [g(y) - f(y)]^2 dy$
- C. $\frac{\pi}{2} \int_c^d [g(y) - f(y)]^2 dy$
- D. $\frac{\pi}{2} \int_a^b [g(x) - f(x)]^2 dx$
- E. None of the above

Question 9

Let R be the region in the first quadrant enclosed by the x -axis, the y -axis, the line $y=2$, and the curve $y=\sqrt{9-x^2}$.

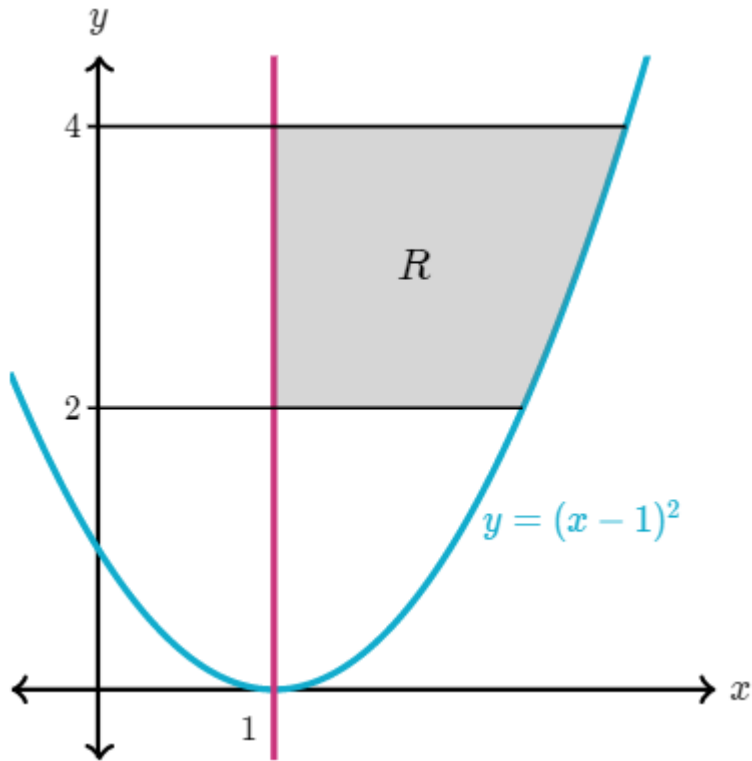


A solid is generated by rotating R about the y -axis. What is the volume of the solid in terms of π ?

- A. $\frac{35\pi}{3}$
- B. $\frac{46\pi}{2}$
- C. $\frac{28\pi}{9}$
- D. $\frac{46\pi}{3}$

Question 10

Let R be the region enclosed by the line $x=1$, the line $y=2$, the line $y=4$, and the curve $y=(x-1)^2$.

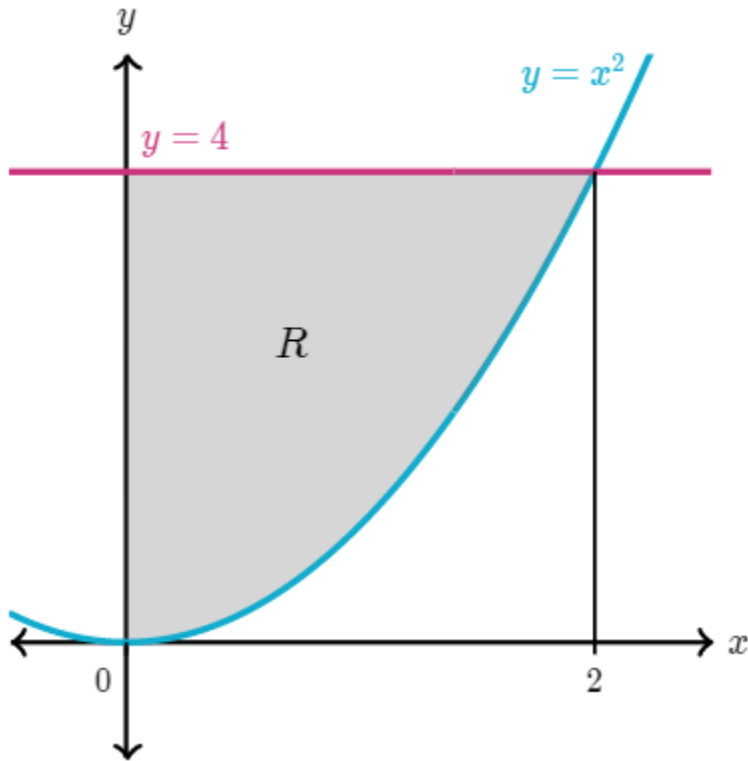


A solid is generated by rotating R about the line $x=1$. What is the volume of the solid in terms of π ?

- A. 4π
- B. 5π
- C. 6π
- D. 7π

Question 11

Region R in the first quadrant is enclosed by the y -axis, the line $y=4$, and the curve $y=x^2$.

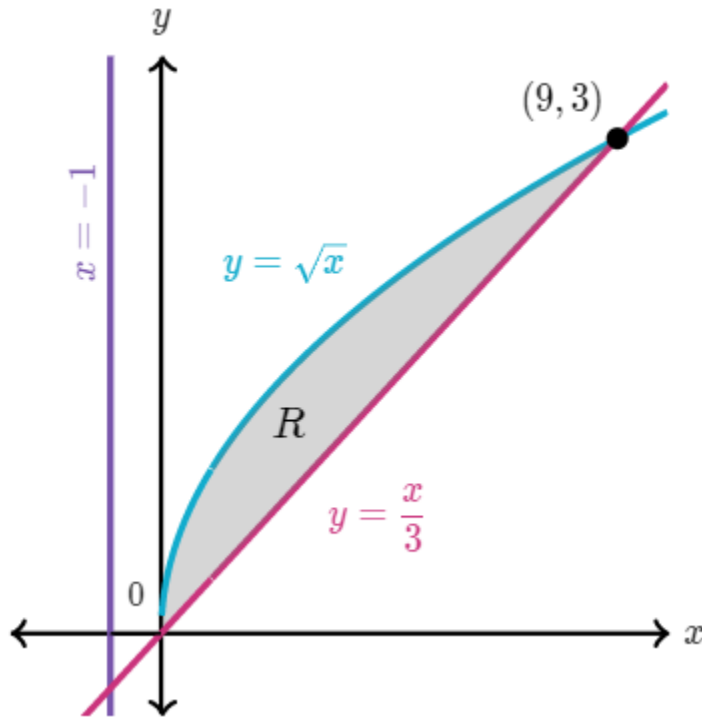


What is the volume of the solid generated when R is rotated about the x -axis in terms of π ?

- A. $\frac{122\pi}{5}$
- B. $\frac{128\pi}{5}$
- C. $\frac{121\pi}{5}$
- D. $\frac{112\pi}{5}$

Question 12

Let R be the region enclosed by the curves $y = \sqrt{x}$ and $y = \frac{x}{3}$.



A solid is generated by rotating R about the line $x = -1$. What is the volume of the solid in terms of π ?

- A. $\frac{207\pi}{5}$
- B. $\frac{208\pi}{5}$
- C. $\frac{209\pi}{5}$
- D. 42π

Question 13

A particle moves along the x -axis with velocity $v(t) = \ln(t^2 + 5t + 1)$.

What is the particle's displacement between the times $t = 1$ and $t = 5$? Round your answer to three decimal places.

- A. ≈ 12.534
- B. ≈ 13.534
- C. ≈ 14.534
- D. ≈ 15.534

Question 14

What is the average value of $\sqrt[3]{x}$ on the interval $-5 \leq x \leq 9$?

- A. $\frac{3}{56} \cdot (\sqrt[4]{9^3} - \sqrt[4]{5^3})$
- B. $\frac{3}{56} \cdot (\sqrt[3]{9^4} - \sqrt[3]{5^4})$ correct answer
- C. $\frac{1}{2} \cdot (\sqrt[3]{9} - \sqrt[3]{-5})$
- D. $\frac{1}{2} \cdot (\sqrt[3]{9} + \sqrt[3]{-5})$

Question 15

Determine the volume of the region bounded by $y = x^2 - 2x$ and $y = x$ that is rotated about $y = 4$.

- A. 5.4
- B. 30.6
- C. 96.133
- D. 108.332

Question 16

What integral would allow you to find the volume of the region bounded by $y = 2x^2$ and $y = 8$ around the line $y = 11$.

- A. $\pi \int_0^2 (4x^4 - 8) dx$
- B. $\pi \int_0^2 [(11 - 2x^2)^2 - 9] dx$
- C. $\pi \int_{-2}^2 [(11 - 2x^2)^2 - 9] dx$ correct answer
- D. $\pi \int_{-2}^2 (4x^4 - 8) dx$

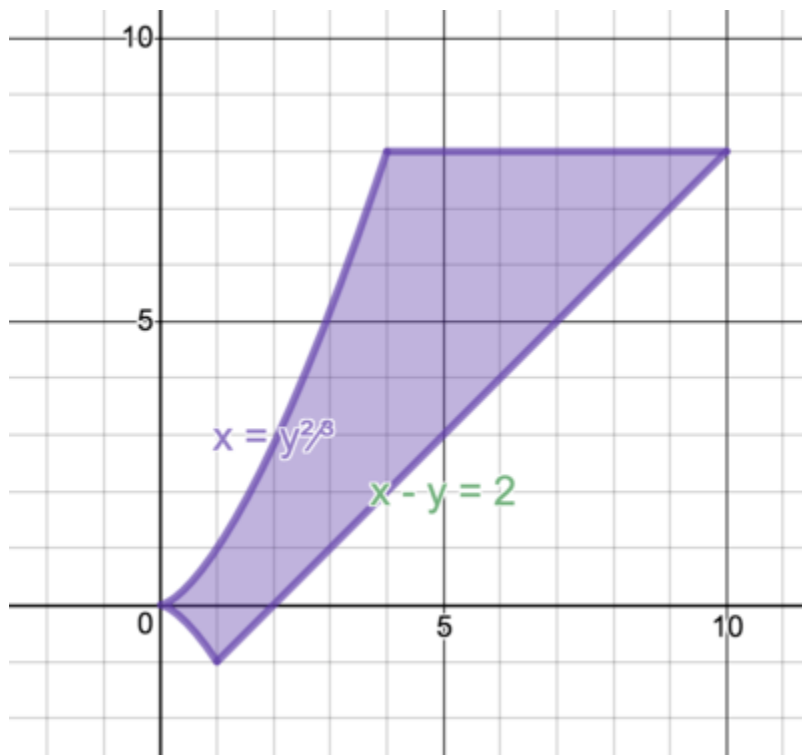
Question 172

If the region enclosed by $y=3x^2$, $y=3$ is revolved about the x -axis, what is the volume of the solid generated?

- A. $\frac{72\pi}{5}$
- B. $\frac{27\pi}{2}$
- C. $\frac{27\pi}{20}$
- D. $\frac{9\pi}{10}$

Question 18

Write the integral that would be used to find the area bounded by the graphs of $x-y=2$, $y=8$ and $x=y^{\frac{2}{3}}$.



A. $\int_{-1}^8 (y + 2 - y^{\frac{2}{3}}) dx$ correct answer

B. $\int_{-1}^8 \left(y^{2/3} - y + 2 \right) dx$

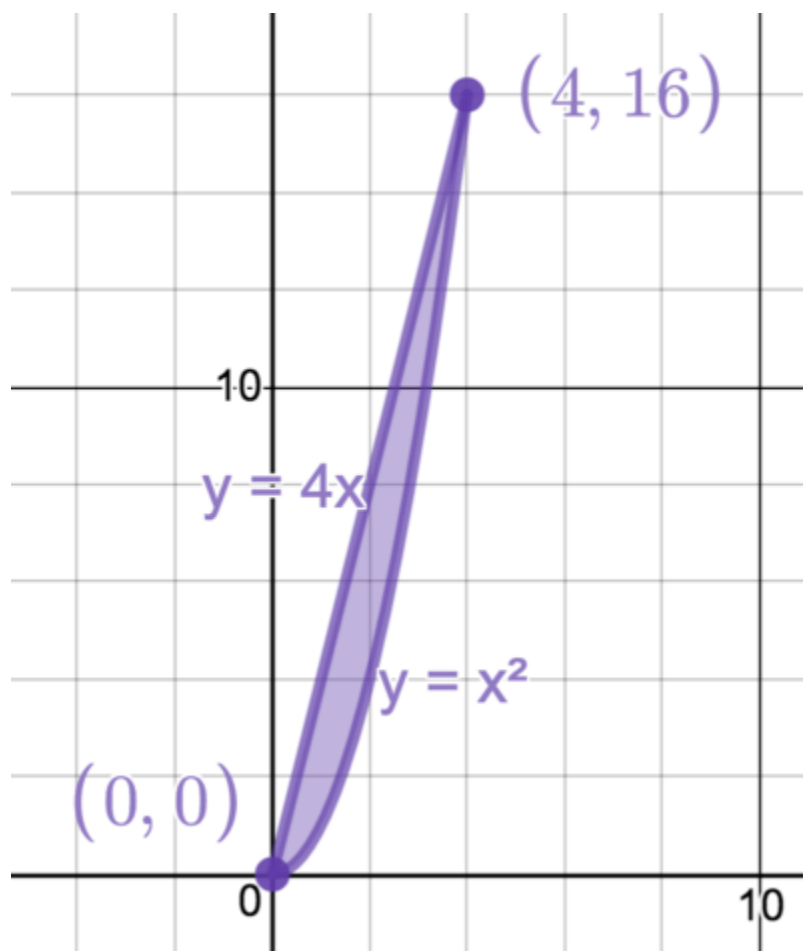
C. $\int_{-1}^8 \left(y^{2/3} - y - 2 \right) dx$

D. $\int_0^{10} \left(8 - x - 2 \right) dx$

E. $\int_0^{10} \left(x^{3/2} - x - 2 \right) dx$

Question 19

Find the area of the region bounded by the graphs of $y=x^2$ and $y=4x$.



- A. $\frac{32}{3}$
- B. $\frac{64}{3}$
- C. 32
- D. 64
- E. $\frac{32}{5}$

Answer Key

1. A
2. D
3. D
4. A
5. A
6. A
7. B
8. A
9. D
10. C
11. B
12. A
13. A
14. B
15. C
16. C
17. A
18. A
19. A